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Systematics of moduli stabilisation in Calabi-Yau flux compactifications

Vijay Balasubramanian

David Rittenhouse Laboratories, University of Pennsylvania
Philapdelphia, PA 19104, U.S.A.
*E-mail:*e-mail:vijay@physics.upenn.edu

Peter Berglund

Department of Physics, University of New Hampshire
Durham, NH 03824, U.S.A.
E-mail: per.berglund@unh.edu

Joseph P. Conlon and Fernando Quevedo

DAMTP, Centre for Mathematical Sciences
Wilberforce Road, Cambridge, CB3 0WA, U.K.
E-mail: j.p.conlon@damtp.cam.ac.uk, f.quevedo@damtp.cam.ac.uk

Abstract: We study the large volume limit of the scalar potential in Calabi-Yau flux compactifications of type IIB string theory. Under general circumstances there exists a limit in which the potential approaches zero from below, with an associated non-supersymmetric AdS minimum at exponentially large volume. Both this and its de Sitter uplift are tachyon-free, thereby fixing all Kähler and complex structure moduli. Also, for the class of vacua described in this paper, the gravitino mass is independent of the flux discretuum, whereas the ratio of the string scale to the 4d Planck scale is hierarchically small but flux dependent. The inclusion of α' corrections plays a crucial role in the structure of the potential. We illustrate these ideas through explicit computations for a particular Calabi-Yau manifold.

Keywords: Superstrings and Heterotic Strings, Superstring Vacua.

Contents

1. Introduction

In [1] an elegant scenario was proposed for fixing complex structure, Kähler structure and dilaton moduli in type IIB Calabi-Yau compactifications. If this scenario is actually realised in explicit models it will address the main obstacle preventing string theory making contact with low energy physics. This scenario has already given rise to many generalisations with applications to realistic models and cosmological inflation. It has also opened new directions regarding naturalness issues in string theory in the context of the string theory landscape. There is some question as to whether such a low-energy effective potential on the string configuration space actually exists (see, e.g., [2]). However, in this paper we will not try to address this subtle issue, but simply investigate the structure of the effective potentials resulting from IIB flux compactifications.

The main ingredients of the KKLT scenario are the presence of fluxes of RR and NS fields [3, 4], responsible for the fixing of the dilaton and complex structure moduli, and non-perturbative effects that fix the Kähler moduli. (See [5] for a sampling of recent work in additional settings.) The minimum of the potential can be lifted to a positive value by additional mechanisms such as adding D-terms through the inclusion of anti-D3 branes [1] or D7-brane magnetic fluxes [6].[1] Model building in this context must contend with several fine-tuning and stability issues. In particular, the superpotential induced by the fluxes must be hierarchically small ($< 10^{-4}$) in order to obtain solutions with large volume in which the effective field theory approximation can be trusted, and the fluxes must fix the dilaton at small string coupling to suppress loop effects. Although the complex structure moduli and dilaton are fixed at a minimum of the potential before the non-perturbative and supersymmetry breaking effects are included, these can destabilize some of the scalars (see,

[1] Another possible way to get positive vacuum energy is through F-breaking terms in the Kähler moduli sector, by considering the flux superpotential to be of $O(1)$ [7]. Additional ways of breaking supersymmetry and achieving a metastable de Sitter minimum appear in [8].

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e.g., [9]). A statistical analysis of the discrete flux choices reveals interesting facts such as a distribution of effective potential extrema that peaks close to conifold points (although this tends to be accompanied by an increase in tachyonic directions in these regions [10]). Because of the dependence on the discrete flux choice, the superpotential and the associated supersymmetry breaking scale are scanned by the different vacua, and we are led to regard these quantities as environmentally (rather than dynamically) determined in our world [11]. Different proposals have been put forward regarding the preferred supersymmetry breaking scale [12]. In addition to the question of whether such a landscape of vacua actually exists in a meaningful sense [2], it is clear from the above that the conclusions from this scenario are highly model dependent.

In the present article we will extract model-independent properties of this class of compactifications by studying the large volume limit for a general model with more than one Kähler and complex structure modulus ($h_{12} > h_{11} > 1$). We will argue that the combination of $(\alpha')^3$ effects and non-perturbative contributions to the superpotential will generically give rise to a large volume non-supersymmetric AdS vacuum, differing from the simplest KKLT scenario in which the AdS minimum is found to be supersymmetric. To reach this conclusion, we show that there is a large volume limit in which the potential goes asymptotically to zero from below, while it is also positive at small volumes. This induces the existence of a large volume AdS minimum.[2]

The non-supersymmetric minimum that we find is at exponentially large volume, and is essentially independent of the value of the flux superpotential. We argue that the non-perturbative effects will not destabilize the flux-stabilized complex structure and dilaton moduli. Supersymmetry is broken by the Kähler moduli only and the gravitino mass is *not* flux-dependent: thus it does not scan from vacuum to vacuum as the fluxes are tuned, but is rather peaked at a particular value for a particular Calabi-Yau. The ratio of the string scale to the 4d Planck scale can be made hierarchically small and its value does depend on the fluxes. All of this substantially changes the general picture described above. We illustrate this behaviour through a particular model, the orientifold of $\mathbb{P}^4_{[1,1,1,6,9]}$.

2. Review of type IIB flux compactifications

The moduli scalars of string theory compactifications can be stabilized by turning on fluxes on the internal manifold (see [4] for reviews). We work in IIB theory compactified on Calabi-Yau orientifolds, with RR and NS-NS 3-form fluxes, denoted by F_3 and H_3 respectively. These are restricted to have integral cohomology in string theory:

$$\frac{1}{(2\pi)^2\alpha'}\int_{\Sigma_a} F_3 = n_a \in \mathbb{Z}, \qquad \frac{1}{(2\pi)^2\alpha'}\int_{\Sigma_b} H_3 = m_b \in \mathbb{Z}\,,$$

where Σ_3 is a 3-cycle in the internal space M. This framework [13] can be viewed as a limit of F-theory compactifications on an elliptically fibered Calabi-Yau fourfold X [14]. The orientifold action results in O3-planes (and wrapped D7-branes/O7-planes) carrying

[2]While the total volume is very large, some moduli could be stabilized near the string scale, depending on the particular Calabi-Yau considered.

an anti-D3 brane charge $\chi(X)/24$ determined by the Euler characteristic χ of the fourfold X. This charge may be cancelled by the addition of D3-branes, or through the effective D3-brane charge induced by F_3 and H_3 fluxes via the the Chern-Simons term in the 10D supergravity action:

$$S_{CS} \sim \int C_4 \wedge F_3 \wedge H_3 \,. \tag{2.1}$$

This term contributes a D3-brane charge $\int_M H_3 \wedge F_3$ leading to the condition

$$N_{D3} - N_{\bar{D}3} + \frac{1}{(2\pi)^4 \alpha'^2} \int H_3 \wedge F_3 = \frac{\chi(X)}{24} \,. \tag{2.2}$$

Internal magnetic fields on the D7-branes may also contribute D3-brane charge, but we will not turn on those fields here. In order to avoid the need to stabilize scalars parametrizing D3-brane positions we will require that the fluxes saturate (2.2).

We will analyze the resulting warped product of a 4d spacetime with an internal space that is conformally Calabi-Yau [13] in the framework of $\mathcal{N}=1$ supergravity. Ignoring gauge sectors, the theory is specified by the Kähler potential and the superpotential. The latter takes the form [15]

$$W = \int_M G_3 \wedge \Omega \,, \tag{2.3}$$

where $G_3 = F_3 - \tau H_3$, with τ being the axion-dilaton field, and Ω the holomorphic $(3,0)$ form of the Calabi-Yau. This does not depend on the Kähler moduli. To leading order in g_s and α', the Kähler potential is given by the Weil-Petersson metric derived by Kaluza-Klein reduction of type IIB supergravity,

$$K_{\text{no-scale}} = -2\log\left[\mathcal{V}\right] - \log\left[-i\int_M \Omega \wedge \bar{\Omega}\right] - \log\left[-i\left(\tau - \bar{\tau}\right)\right], \tag{2.4}$$

where $\mathcal{V}$ is the classical volume of M in units of $l_s = (2\pi)\sqrt{\alpha'}$. This Kähler potential is well-known to possess no-scale structure. Thus, in the $\mathcal{N}=1$ supergravity scalar potential,

$$V = e^K \left[G^{i\bar{j}} D_i W \bar{D}_j \bar{W} - 3|W|^2\right], \tag{2.5}$$

where i, j run over all the moduli, the sum over Kähler moduli cancels the $3|W|^2$ term, with the resulting potential being

$$V_{\text{no-scale}} = e^K G^{a\bar{b}} D_a W \bar{D}_b \bar{W} \,, \tag{2.6}$$

where a and b run over dilaton and complex-structure moduli only. As $V_{\text{no-scale}}$ is positive definite, we can locate the complex structure moduli at a minimum of the potential by solving

$$D_a W = 0 \equiv \partial_a W + (\partial_a K) W = 0 \,. \tag{2.7}$$

This can be done for generic choices of the fluxes and we denote the value of W following this step as W_0.

Since the fluxes are specified by their cohomology, while typical Calabi-Yau manifolds have $h^3 = \mathcal{O}(200)$ 3-cycles, there are many discrete flux choices that satisfy the consistency condition (2.2). This renders the study of complex structure moduli stabilization amenable

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to statistical analysis. This has been carried out by Douglas and collaborators in a series of beautiful papers [10]. These analytic results have been successfully compared with numerical studies of moduli stabilisation using Monte Carlo simulations to generate fluxes [16, 17] (see also [18]), and the statistical approach has been extended to other questions [19, 20]. Here we merely summarise these results.

First, the number density of vacua stabilised near a point z in complex structure moduli space is

$$I_{AD}(z) \sim \left(\frac{\chi}{24}\right)^{(2h^{2,1}+2)} \det(-\mathcal{R} - \omega)\,, \tag{2.8}$$

where $\mathcal{R}$ is the curvature two-form on complex structure moduli space. This formula is the quantitative basis for the claim that there exists exponentially many flux vacua. The determinant has a simple form but a rich structure, for example showing that vacua cluster near conifold loci.

We will make most use of the result that in the discretuum of vacua arising from flux choices, the values of $e^{K_{cs}}|W_0|^2$ are uniformly distributed. Here K_{cs} refers to the dilaton and complex-structure dependent parts of (2.4). This has a similar form to the gravitino mass $m_{\frac{3}{2}} = e^{K/2}|W|$, but as the full Kähler potential K is volume dependent, the physical gravitino mass depends on the stabilized value of the volume modulus.

In this article we will study how the leading corrections to the Kähler potential and the superpotential, perturbative for the former and non-perturbative for the latter, affect the structure of the scalar potential. For the features we uncover the subleading corrections have subleading effects and can be consistently ignored.

The leading corrections to the Kähler potential were computed in [21] and arise from an $\mathcal{O}(\alpha'^3)$ term similar to that appearing in type II compactifications. (See also [22].) Measuring dimensionful quantities in units of $l_s = 2\pi\sqrt{\alpha'}$, the resulting Kähler potential takes the form

$$K_{\alpha'} = -2\log\left[e^{-3\phi_0/2}\mathcal{V} + \frac{\xi}{2}\left(\frac{-i\,(\tau-\bar\tau)}{2}\right)^{3/2}\right] - \log\left[-i\int_M \Omega\wedge\bar\Omega\right] - \log\left[-i\,(\tau-\bar\tau)\right], \tag{2.9}$$

where $\xi = -\frac{\zeta(3)\chi(M)}{2(2\pi)^3}$. We will require $\xi > 0$, which is equivalent to $h^{2,1} > h^{1,1}$: i.e., more complex structure than Kähler moduli. This can be compared with the analogous, exact, correction in pure type II compactifications

$$K = -2\log\left[\mathcal{V} + \frac{\xi}{2} + \text{ worldsheet instantons}\right]. \tag{2.10}$$

Although the internal volume is measured in units of l_s, the $\mathcal{N}=1$ supergravity potential is in units of M_{pl}. This arises from the string theoretic starting point through a standard Weyl rescaling to 4-d Einstein frame. The dilaton dependence in (2.9) modifies the Kähler metric such that mixed $G^{\tau\bar\rho_i}$ terms no longer vanish. However, as was shown in [7], for large volume we note that the behaviour described below is insensitive to the difference between the corrections in (2.9) and (2.10).[3]

[3]A possible subtlety here is that the field strengths that we have turned on could give rise to additional

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The superpotential is not renormalised at any order in perturbation theory and receives no α' corrections. However, under certain circumstances it acquires a non-perturbative dependence on some or all of the Kähler moduli through D3-brane instantons [23] or gaugino condensation from wrapped D7-branes [24]. It then takes the form

$$W = \int_M G_3 \wedge \Omega + \sum_i A_i e^{i a_i \rho_i} \,, \tag{2.11}$$

where A_i is a one-loop determinant. For D3-brane instantons, A_i only depends on the complex structure moduli. Here $a_i = \frac{2\pi}{K}$ with $K \in \mathbb{Z}_+$ and $K = 1$ for D3-instantons, while $\rho_i \equiv b_i + i\tau_i$ are the complexified Kähler moduli, with τ_i the four-cycle modulus, which is the volume of the divisor $D_i \in H_4(M, \mathbb{Z})$, given by

$$\tau_i = \partial_{t_i} \mathcal{V} = \frac{1}{2} \kappa_{ijk} t^j t^k \,. \tag{2.12}$$

Here the Kähler class is given by $J = \sum_i t^i D_i$ (by Poincare' duality $D_i \in H^2(M, \mathbb{Z})$), with the t^i measuring the areas of 2-cycles and the classical volume being

$$\mathcal{V} = \int_M J^3 = \frac{1}{6} \kappa_{ijk} t^i t^j t^k \,. \tag{2.13}$$

We should understand $\mathcal{V}$ as an implicit function of the the complexified 4-cycle moduli ρ_k via the relation between τ_i and the t^i in (2.12).

Equations (2.9) and (2.11) completely specify the theory. However, trying to directly visualise the full potential is not illuminating. We therefore follow KKLT [1] and first integrate out the complex structure moduli. Technically, we stabilise the dilaton and complex structure moduli through solving (2.7), and then regard their values as fixed. This leaves a theory only depending on Kähler moduli, which we then stabilise separately. It is important to ask whether the resulting critical point of the full potential is genuinely a minimum or merely a saddle point. For the simplest implementation of the KKLT scenario with one Kähler modulus and a rigid Calabi-Yau, the resulting potential has no minima [9]. However, one can argue that true minima can occur in many-modulus models [28]. We shall show below that the vacua we find, whether AdS or uplifted dS, are automatically tachyon-free.

After integrating out the dilaton and complex structure moduli the Kähler and superpotential become

$$K = K_{cs} - 2 \log \left[e^{-\frac{3\phi_0}{2}} \mathcal{V} + \frac{\xi}{2} \left(\frac{-i(\tau - \bar{\tau})}{2} \right)^{3/2} \right] , \tag{2.14}$$

$$W = W_0 + \sum_n A_n e^{i a_n \rho_n} \,. \tag{2.15}$$

The dilaton-dependent terms in the Kähler moduli dependent part of K are shown as they are necessary to determine correctly the form of the inverse metric although we then regard the dilaton as fixed by the fluxes. We note here for subsequent use that as $\mathcal{V} \to \infty$ the

complex structure dependent corrections to the Kähler potential (2.9). Still, by dimensional analysis we expect that such contributions are relatively suppressed at large volume. (See the discussion section.) We thank Arvind Rajaraman for useful discussions on this point.

Kähler potential behaves as

$$e^K \sim \frac{e^{K_{cs}}}{\mathcal{V}^2} + \mathcal{O}\left(\frac{1}{\mathcal{V}^3}\right), \tag{2.16}$$

where all dilaton-dependent terms have been absorbed into $e^{K_{cs}}$. If we substitute (2.14) and (2.15) into equation (2.5), we obtain the following potential [7]:

$$\begin{aligned} V &= e^K \Bigg[G^{\rho_j \bar{\rho}_k} \left(a_j A_j a_k \bar{A}_k e^{i(a_j \rho_j - a_k \bar{\rho}_k)} + i \left(a_j A_j e^{i a_j \rho_j} \bar{W} \partial_{\bar{\rho}_k} K - a_k \bar{A}_k e^{-i a_k \bar{\rho}_k} W \partial_{\rho_j} K \right) \right) + \\ &\qquad + 3\xi \frac{(\xi^2 + 7\xi\mathcal{V} + \mathcal{V}^2)}{(\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} |W|^2 \Bigg] \qquad (2.17) \\ &\equiv V_{np1} + V_{np2} + V_{\alpha'} \,. \qquad (2.18) \end{aligned}$$

This potential has one well-known class of minima, namely the KKLT solution [1]. This requires a small value of W_0 (typically $< 10^{-4}$), obtained by tuning fluxes, and the existence of a non-perturbative superpotential depending on all Kähler moduli. The supersymmetry conditions

$$D_{\rho_i} W = 0 \tag{2.19}$$

may then be solved to stabilise all Kähler moduli at a large-volume supersymmetric AdS minimum, which may or may not be a minimum of the full potential. In [25] some Calabi-Yaus are explicitly constructed for which an appropriate non-perturbative superpotential will be generated, and a direct realisation of the KKLT scenario should be possible for the models described there. There are however two disadvantages to the KKLT scenario. The first is that one must explicitly check whether a particular solution is a minimum of the full potential. The second is the small value of W_0 required, given the statistical results on the flux distribution of $e^K|W_0|^2$. Of course, vast numbers of flux vacua still imply vast-but-not-quite-so-vast numbers of flux choices with appropriate values of W_0. Nonetheless, given that in the flux 'landscape' values of W_0 of $\mathcal{O}(10)$ are more common than those of order $\mathcal{O}(10^{-4})$ by a factor $\sim 10^{10}$, it would be interesting to have examples of large-volume minima of the potential for large values of W_0. We now turn our attention to this.

3. Large volume limit

In section 3.1 we study the large-volume limit of the potential (2.17) for a general Calabi-Yau manifold. In section 3.2 we shall then illustrate our ideas through explicit computations on a particular orientifold model.

3.1 The general case

The argument for a large-volume AdS minimum of the potential (2.17) has two stages. We first show that there will in general be a decompactification direction in moduli space along which

1. The divisor volumes $\tau_i \equiv \mathrm{Im}(\rho_i) \to \infty$.

2. $V < 0$ for large $\mathcal{V}$, and thus the potential approaches zero from below.

This leads to an argument that there must exist a large volume AdS vacuum. It might have been expected that the positive $(\alpha')^3$ term, scaling as $+\frac{1}{\mathcal{V}^3}$, will dominate at large volume over the non-perturbative terms which are exponentially suppressed. However, care is needed: the $(\alpha')^3$ term is perturbative in the volume of the entire Calabi-Yau, whereas the naively suppressed terms are exponential in the divisor volumes separately. Hence, in a large volume limit in which some of the divisors are relatively small the non-perturbative terms can compete with the peturbative ones.

The $(\alpha')^3$ term in (2.17) denoted by $V_{\alpha'}$ is easiest to analyse in the large $\mathcal{V}$ limit. Owing to the large volume behaviour (2.16) of the Kähler potential, this scales as

$$V_{\alpha'} \sim +\frac{3\xi}{16\mathcal{V}^3} e^{K_{cs}} |W|^2 + \mathcal{O}\left(\frac{1}{\mathcal{V}^4}\right). \tag{3.1}$$

We observe that $V_{\alpha'}$ is always positive and depends purely on the overall volume $\mathcal{V}$.

However, V_{np1} and V_{np2} both depend explicitly on the Kähler moduli and we must be more precise in specifying the decompactification limit. We consider the $\mathcal{V} \to \infty$ limit in moduli space where $\tau_i \to \infty$ for all moduli except one, which we denote by τ_s. There are two conditions on τ_s. The first is that this limit is well-defined; for example, the volume should not become formally negative in this limit. The second is that τ_s must appear non-perturbatively in W, preferably through D3-instanton effects. It is however not essential for this purpose that all Kähler moduli appear non-perturbatively in the superpotential.

Let us now study V_{np1} in this limit. From (2.17),

$$V_{np1} = e^K G^{\rho_j \bar{\rho}_k} \left(a_j A_j a_k \bar{A}_k e^{i(a_j \rho_j - a_k \bar{\rho}_k)}\right). \tag{3.2}$$

This will be positive definite. As we have taken τ_i large for $i \neq s$, the only term not exponentially suppressed in (3.2) is that involving ρ_s alone. V_{np1} then reduces to

$$V_{np1} = e^K G^{\rho_s \bar{\rho}_s} a_s^2 |A_s|^2 e^{-2a_s \tau_s}. \tag{3.3}$$

We need to determine the inverse metric $G^{\rho_i \bar{\rho}_j}$. With α' corrections included, this is given by (e.g. see [29])[4]

$$G^{\rho_i \bar{\rho}_j} = -\frac{2}{9}\left(2\mathcal{V} + \xi\right) k_{ijk} t^k + \frac{4\mathcal{V} - \xi}{\mathcal{V} - \xi} \tau_i \tau_j. \tag{3.4}$$

In the large $\mathcal{V}$ limit this becomes

$$G^{\rho_i \bar{\rho}_j} = -\frac{4}{9} \mathcal{V} k_{ijk} t^k + 4\tau_i \tau_j + \text{ (terms subleading in } \mathcal{V}). \tag{3.5}$$

Thus in the limit described above we have

$$G^{\rho_s \bar{\rho}_s} = -\frac{4}{9} \mathcal{V} k_{ssk} t^k + \mathcal{O}(1), \tag{3.6}$$

with

$$V_{np1} \sim \frac{(-k_{ssk} t^k) a_s^2 |A_s|^2 e^{-2a_s \tau_s} e^{K_{cs}}}{\mathcal{V}} + \mathcal{O}\left(\frac{e^{-2a_s \tau_s}}{\mathcal{V}^2}\right). \tag{3.7}$$

[4]The conventions used for the Kähler moduli in [29] differ slightly from ours; as we are in this section only interested in the overall sign of the potential these are not important.

Here we have dropped numerical prefactors. Despite the minus sign in front of (3.6) this component of the inverse metric will be positive since the Kähler metric as a whole is positive definite and this component computes the length squared of the (dual) vector $\partial_{\rho_s} W$. In the limit we are considering, so long as we remain inside the Kähler cone, the leading term which we keep must be positive.

We can perform a similar analysis for V_{np2}, from whence negative contributions to the potential arise. We have

$$V_{np2} = +ie^K \left(G^{\rho_j \bar{\rho}_k} a_j A_j e^{i a_j \rho_j} \bar{W} \partial_{\bar{\rho}_k} K - G^{\rho_k \bar{\rho}_j} a_k \bar{A}_k e^{-i a_k \bar{\rho}_k} W \partial_{\rho_j} K \right). \tag{3.8}$$

The only surviving exponential terms are again those involving τ_s. V_{np2} thus reduces to

$$V_{np2} = e^K \left[G^{\rho_s \bar{\rho}_k} \left(i a_s A_s e^{i a_s \rho_s} \bar{W} \partial_{\bar{\rho}_k} K \right) + G^{\rho_k \bar{\rho}_s} \left(-i a_s \bar{A}_s e^{-i a_s \bar{\rho}_s} W \partial_k K \right) \right]. \tag{3.9}$$

The form of the inverse metric (3.4) implies $G^{\rho_s \bar{\rho}_k} = G^{\rho_k \bar{\rho}_s}$. The sign of V_{np2} is determined by the value of the axionic field $b_s = \mathrm{Re}(\rho_s)$, which will adjust itself to make V_{np2} negative. To see this, note first that at leading order in the large volume limit we are considering $W = W_0 + O(1/\mathcal{V})$, so that the only dependence on the axion b_s is in V_{np2}. Now write $V_{np2} = e^{i a_s b_s} X + e^{-i a_s b_s} \bar{X}$ where we have collected all factors in (3.9) except for the axion into X and $\bar{X}$. Extremizing the potential with respect to b_s, it is easy to see that at a minimum the axion will arrange its value so as to cancel the overall phase from the prefactors to make V_{np2} negative.[5]

Thus we may without loss of generality simplify the calculation by replacing ρ_s by $i\tau_s$ and assuming A_s and W to be both real. Recall that

$$\partial_{\rho_k} K = \frac{i t^k}{2\mathcal{V} + \xi} \rightarrow \frac{i t^k}{2\mathcal{V}} + \mathcal{O}\left(\frac{1}{\mathcal{V}^2}\right).$$

Therefore

$$V_{np2} \sim -e^K a_s A_s W e^{-a_s \tau_s} G^{\rho_s \bar{\rho}_j} \frac{t^j}{\mathcal{V}} + \mathcal{O}\left(\frac{e^{-a_s \tau_s}}{\mathcal{V}^2}\right). \tag{3.10}$$

We have introduced a minus sign as a reminder that this term will be negative. Substituting in for $G^{\rho_s \bar{\rho}_k}$ then gives

$$\begin{aligned} V_{np2} &\sim -e^{K_{cs}} a_s A_s W e^{-a_s \tau_s} \frac{-\frac{4}{9}\mathcal{V} k_{sjk} t^j t^k + 4\tau_s \tau_j t^j}{\mathcal{V}^3} + \mathcal{O}\left(\frac{e^{-a_s \tau_s}}{\mathcal{V}^3}\right) \\ &\sim -e^{K_{cs}} a_s A_s W e^{-a_s \tau_s} \frac{-\frac{8}{9}\mathcal{V} \tau_s + 4\tau_s \tau_j t^j}{\mathcal{V}^3} + \mathcal{O}\left(\frac{e^{-a_s \tau_s}}{\mathcal{V}^3}\right). \end{aligned}$$

From the definition of τ_i, the $\tau_s \tau_j t^j$ term will behave as $\tau_s \mathcal{V}$ giving a uniform volume scaling in the numerator. Then we conclude that in the limit described above,

$$V_{np2} \sim -\frac{a_s \tau_s e^{-a_s \tau_s}}{\mathcal{V}^2} |A_s W_0| e^{K_{cs}} + \mathcal{O}\left(\frac{e^{-a_s \tau_s}}{\mathcal{V}^3}\right). \tag{3.11}$$

[5] It is interesting to note that the phase due to b_s changes sign between the supersymmetric solution, as found by Denef et al [25], and the non-supersymmetric solution discussed here. This also occurs for the non-supersymmetric vacua of the type discussed in [7] that exist in this model for $W_0 = O(1)$ at volumes only slightly larger than the string scale. We thank Kevin Rehberg for making this observation.

We may now study the full potential by combining equations (3.1), (3.7) and (3.11).

$$V \sim \left[\frac{1}{\mathcal{V}}a_s^2|A_s|^2(-k_{ssk}t^k)e^{-2a_s\tau_s} - \frac{1}{\mathcal{V}^2}a_s\tau_s e^{-a_s\tau_s}|A_sW| + \frac{\xi}{\mathcal{V}^3}|W|^2\right]. \tag{3.12}$$

We have absorbed factors of $e^{K_{cs}}$ into the values of W and A_s. There exists a particular decompactification limit in which this potential approaches zero from below. This limit is given by

$$\mathcal{V} \to \infty \qquad \text{with} \qquad a_s\tau_s = \ln(\mathcal{V})\,. \tag{3.13}$$

In this limit the potential takes the following form

$$V \sim \left[a_s^2A_s^2\frac{(-k_{ssj}t^j)}{\mathcal{V}^3} - |A_sW_0|\frac{(a_sk_{sjk}t^jt^k)}{\mathcal{V}^3} + \frac{\xi}{\mathcal{V}^3}|W_0|^2\right] + \mathcal{O}\left(\frac{1}{\mathcal{V}^4}\right). \tag{3.14}$$

As in this limit the non-perturbative corrections to W are subleading in $\mathcal{V}$, we have replaced W by W_0. We have also written out $\tau_s = \frac{\ln\mathcal{V}}{a_s}$ in terms of 2-cycle volumes t^i.

All terms in equation (3.14) have the same volume dependence and it is not immediately obvious which is dominant at large volume. However, the numerator of the second term of equation (3.14) is quadratic in the 2-cycle volumes, whereas the others have at most a linear dependence. As $\tau_i \to \infty$ for all i, all 2-cycles must blow up to infinite volume. The numerator of the second term is proportional to the volume of the τ_s 4-cycle (2.12) and thus scales as $\ln\mathcal{V}$. Thus, this term scales as $\frac{\ln\mathcal{V}}{\mathcal{V}^3}$, and overcomes the first and third terms which scale schematically as $\frac{\sqrt{\ln\mathcal{V}}}{\mathcal{V}^3}$ and $\frac{1}{\mathcal{V}^3}$. In the limit (3.13) the potential behaves as

$$V \sim -e^{K_{cs}}|A_sW_0|\frac{\ln\mathcal{V}}{\mathcal{V}^3}, \tag{3.15}$$

and approaches zero from below.[6]

Given this, it is straightforward to argue that there must exist a large volume AdS minimum. At smaller volumes, the dominant term in the potential (3.12) is either the non-perturbative term V_{np1} or the $(\alpha')^3$ term $V_{\alpha'}$, depending on the value of τ_s. Both are positive; the former because the metric on moduli space is positive definite and the latter because we have required $h^{2,1} > h^{1,1}$. Thus at small volumes the potential is positive, and so since the potential must go to zero at infinity and is known to have negative values at finite volume, there must exist a local AdS minimum along the direction in Kähler moduli space where the volume changes.

One may worry that this argument involves the behaviour of the potential at small values of the volume where the α' expansion cannot be trusted. However, for minima at very large volume, the 'small' volumes required in the above argument are still large in

[6]One concern in the above analysis may be our treatment of A_s, which we have treated as a constant. If the A_s were to depend on the Kähler moduli, this may invalidate the argument. However, we are most interested in superpotentials generated through D3-brane instantons for which A_s depends only on the complex structure moduli. In this case we can consistently treat it as constant when considering solely the Kähler moduli. For gaugino condensation the prefactor will generically depend on all moduli. However, even if $A_s \propto V^\gamma$, the above argument carries through if we now take $a_s\tau_s = (\gamma+1)\ln\mathcal{V}$, although polynomial dependence on the Kähler moduli is unlikely to occur due to the combination of holomorphy and shift symmetry - we thank Liam McAllister for discussions on this point.

string units, and we can self-consistently neglect terms of higher order in α'. The relative strength of the α' correction varies between Calabi-Yaus depending on the precise details of the geometry. For the explicit example studied in the next section, the 'small' volumes used in the above argument to establish the positivity of the potential may be $\mathcal{O}(10^8)$ in string units.

It remains to argue that the potential also has a minimum in the remaining directions of the moduli space. Imagine moving along the surfaces in the moduli space that are of fixed Calabi-Yau volume, $\mathcal{V}$. Then, as one approaches the walls of the Kähler cone the first term in (3.12) dominates, since it has the fewest powers of volume in the denominator and since the exponential contributions of the moduli that are becoming small cannot be neglected. (Only the exponential contribution of τ_s is given in (3.12) because of the assumed limit, but it is easy to convince oneself that a similar term will appear for any modulus that is small while the overall volume is large.) Thus at large overall volume we expect the potential to grow in the positive direction towards the walls of the Kähler cone, provided all the moduli appear in the non-perturbative superpotential. All told, the potential is negative along the special direction in moduli space that we have described and eventually rises to be positive or to vanish in all other directions. So we expect an AdS minimum.[7] Since $V \sim O(1/\mathcal{V}^3)$ at the minimum, while $e^K|W|^2 \sim O(1/\mathcal{V}^2)$ it is clear that $D_{\rho_s}W \neq 0$ and hence the minimum will be non-supersymmetric

We can heuristically see that the minimum we are arguing for can be at exponentially large volume. The naive measure of the location of the minimum is the value of the volume at which the second term of equation (3.14) becomes dominant. As this occurs when $\ln\mathcal{V}$ is large, we expect to be able to find vacua at large values of $\ln\mathcal{V}$. We will see this explicitly in the example studied below.

We can see that the gravitino mass for these vacua will be independent of the flux choice. If the minimum exists at large volume, it is found by playing off the three terms in equation (3.12) against each other. If we write $\tilde{\mathcal{V}} = \frac{A_s\mathcal{V}}{W_0}$, then (3.12) becomes

$$V \sim \left(\frac{A_s^3}{W_0}\right)\left[\frac{1}{\tilde{\mathcal{V}}}a_s^2(-k_{ssk}t^k)e^{-2a_s\tau_s} - \frac{1}{\tilde{\mathcal{V}}^2}a_s\tau_s e^{-a_s\tau_s} + \frac{\xi}{\tilde{\mathcal{V}}^3}\right]. \tag{3.16}$$

The minimum of this potential as a function of $\tilde{\mathcal{V}}$ is thus independent of A_s and W_0 and, given a_s, depends only on the Calabi-Yau. We therefore have

$$\mathcal{V} \sim \frac{W_0}{A_s} f(a_s, \mathcal{M}) + (\text{subleading corrections})\,, \tag{3.17}$$

where f is a function of the geometry. The gravitino mass is then given by

$$m_{\frac{3}{2}} = e^{\frac{K}{2}}|W| \approx \frac{A_s}{2f(a_s,\mathcal{M})}\,. \tag{3.18}$$

The number of moduli stabilised depends on the number of non-perturbative contributions to the superpotential. If all Kähler moduli appear non-perturbatively in the superpotential, then the dilaton, complex structure and Kähler moduli are all stabilised. If

[7]Of course, given the high dimensionality of the moduli space, this argument is heuristic. In the next section we will give an explicit example.

some of the Kähler moduli, ρ_k, do not appear in the superpotential, then as their axionic parts, b_k, make no contribution to the scalar potential, the b_k at least remain unstabilised. However the volume modulus will be fixed and the flux-invariant behaviour of the gravitino mass will remain.

Although we have just considered the Kähler moduli to find this minimum, it is straightforward to see that it must actully be a minimum of the full potential. Reinstating the dilaton-axion and complex structure moduli, this can then be written [21]

$$\begin{aligned} V = {} & e^K \left(G^{a\bar{b}} D_a W \bar{D}_b \bar{W} + G^{\tau\bar{\tau}} D_\tau W \bar{D}_\tau \bar{W} \right) + e^K \frac{\xi}{2\mathcal{V}} (W \bar{D}_\tau \bar{W} + \bar{W} D_\tau W) + \\ & + V_{\alpha'} + V_{np1} + V_{np2} \,. \end{aligned} \tag{3.19}$$

Recall that the moduli values found above give rise to a negative value of the potential of $\mathcal{O}(\frac{1}{\mathcal{V}^3})$. The first term in (3.19) is positive definite and of $(\frac{1}{\mathcal{V}^2})$. This vanishes iff $D_\tau W = D_{\phi_i} W = 0$. Therefore, any movement of either the dilaton or complex structure moduli away from their stabilised values would create a positive term of $\mathcal{O}(\frac{1}{\mathcal{V}^2})$, which the negative term cannot compete with. Thus this must increase the potential and therefore the solution above automatically represents a minimum of the full potential.

It is instructive to compare this with what happens for KKLT solutions. The scalar potential is

$$V = e^{K_{cs}} \left(\frac{G^{\mu\bar{\nu}} D_\mu W \bar{D}_\nu \bar{W}}{\mathcal{V}^2} - 3\frac{|W|^2}{\mathcal{V}^2} \right). \tag{3.20}$$

If $D_\mu W = 0$ for all moduli, the potential is negative at $\mathcal{O}\left(\frac{1}{\mathcal{V}^2}\right)$. However, if we move one modulus, for concreteness the dilaton, away from its stabilised value, the resulting positive definite contribution $e^{K_{cs}} \frac{G^{\tau\bar{\tau}} D_\tau W \bar{D}_\tau \bar{W}}{\mathcal{V}^2}$ is only of the same order as the minimum. Moving the dilaton alters the value of $e^{K_{cs}}$ and thus may increase the numerator of the negative term. As the positive and negative contributions are of the same order, we see that depending on the magnitude of $G^{\tau\bar{\tau}} D_\tau W \bar{D}_\tau \bar{W}$, this may in general decrease the overall value of the potential. Therefore it is necessary to check explicitly for each choice of fluxes that the resulting potential has no minimum.

The above solution can be uplifted to a de Sitter vacuum through the usual mechanisms of adding anti-D3 branes [1] or turning on magnetic fluxes on D7-branes [6]. For concreteness we take the uplift potential to be

$$V_{\text{uplift}} = +\frac{\epsilon}{\mathcal{V}^2} \,. \tag{3.21}$$

When $\epsilon = 0$, the above minimum still exists and there are many values of the moduli for which $V < 0$. For ϵ sufficiently large, the minimum is entirely wiped out and the potential is positive for all values of the moduli. At a critical value of ϵ the minimum will pass through zero, which by construction is still a minimum of the full potential. Following the arguments in [1] it should be possible to tune ϵ in small steps. After adding the uplift terms, the total potential will go to zero from above at large volumes because (3.21) will overwhelm the $O(1/\mathcal{V}^3)$ negative terms even in the special limit that we have been studying. Hence metastable de Sitter vacua will be achievable.

3.2 Explicit calculations for the orientifold of $P^4_{[1,1,1,6,9]}$

We now illustrate the ideas of section (3.1) through explicit calculations for flux compactifications on an orientifold of the Calabi-Yau manifold given by the degree 18 hypersurface in $\mathbb{P}^4_{[1,1,1,6,9]}$. This has been studied by Denef, Douglas and Florea [25] following earlier work in [27]. The defining equation is

$$z_1^{18} + z_2^{18} + z_3^{18} + z_4^3 + z_5^2 - 18\psi z_1 z_2 z_3 z_4 z_5 - 3\phi z_1^6 z_2^6 z_3^6 = 0\,, \tag{3.22}$$

with $h^{1,1} = 2$ and $h^{2,1} = 272$. The complex structure moduli ψ and ϕ that have been written in (3.22) are those two moduli left invariant under the $\Gamma = \mathbb{Z}_6 \times \mathbb{Z}_{18}$ action whose quotient gives the Greene-Plesser construction of the mirror manifold [26]. There are another 270 terms not invariant under Γ which have not been written explicitly, although some will be projected out by the orientifold action.

We first stabilise the complex structure moduli through an explicit choice of fluxes. We must solve

$$D_\tau W_{cs} = 0 \qquad \text{and} \qquad D_{\phi_i} W_{cs} = 0\,, \tag{3.23}$$

where W_{cs} refers to the flux superpotential, for the dilaton and complex structure moduli. There are two possibilities. First, we may of course turn on fluxes along all relevant three-cycles and solve (3.23) for all moduli. As we would need to know all 200-odd periods this is impractical. We however know of no theoretical reasons not to do it. The easier approach is to turn on fluxes only along cycles corresponding ψ and ϕ, and then solve

$$D_\tau W_{cs} = D_\psi W_{cs} = D_\phi W_{cs} = 0\,. \tag{3.24}$$

As described in [30], the invariance of (3.22) under Γ then ensures that $D_{\phi_k} W = 0$ for all other moduli ϕ_k. The necessary periods have been computed in [27] and appropriate fluxes and solutions to (3.24) could be found straightforwardly along the lines of [16, 17, 30]. This is not our focus in this paper and we henceforth assume this to have been done.

We now return to the Kähler moduli. The Kähler geometry is specified by

$$\mathcal{V} = \frac{1}{9\sqrt{2}} \left(\tau_5^{\frac{3}{2}} - \tau_4^{\frac{3}{2}} \right) \tag{3.25}$$

with

$$\tau_4 = \frac{t_1^2}{2} \qquad \text{and} \qquad \tau_5 = \frac{(t_1 + 6t_5)^2}{2}\,.$$

Here τ_4 and τ_5 are volumes of the divisors D_4 and D_5, corresponding to a particular set of 4-cycles, and t_1 and t_5 2-cycle volumes. Generally the volume is only an implicit function of τ_i, but here we are fortunate and have an explicit expression. As shown in [25], both D_4 and D_5 correspond to divisors which when lifted to a Calabi-Yau fourfold X have arithmetic genus 1 and thus appear non-perturbatively in the superpotential. We write this superpotential as

$$W = W_0 + A_4 e^{i a_4 \rho_4} + A_5 e^{i a_5 \rho_5}\,. \tag{3.26}$$

We now take the limit described in section 3.1, in which $\mathcal{V} \to \infty$ (and hence $\tau_5 \to \infty$) and $\tau_4 \sim \log \mathcal{V}$. Note that the alternative limit $\tau_4 \to \infty$ with $\tau_5 \sim \log \mathcal{V}$ would not be well-defined, as the volume of the Calabi-Yau becomes formally negative.

The α' correction is given by equation (3.1). For V_{np1} and V_{np2} we must compute the inverse metric, which in this limit is given by

$$G^{\rho_4\bar{\rho}_4} = 24\sqrt{2}\sqrt{\tau_4}\mathcal{V} \sim \sqrt{\tau_4}\mathcal{V}\,, \tag{3.27}$$

$$G^{\rho_4\bar{\rho}_5} = G^{\rho_5\bar{\rho}_4} = 4\tau_4\tau_5 \sim \tau_4\mathcal{V}^{\frac{2}{3}}\,, \tag{3.28}$$

$$G^{\rho_5\bar{\rho}_5} = \frac{4}{3}\tau_5^2 \sim \mathcal{V}^{\frac{4}{3}}\,. \tag{3.29}$$

We can then compute V_{np1} and V_{np2} with the result that the full potential takes the schematic form

$$V \sim \left[\frac{1}{\mathcal{V}}a_4^2|A_4|^2\sqrt{\tau_4}e^{-2a_4\tau_4} - \frac{1}{\mathcal{V}^2}a_4\tau_4 e^{-a_4\tau_4}|A_4 W| + \frac{\xi}{\mathcal{V}^3}|W|^2\right]. \tag{3.30}$$

where numerical coefficients have been dropped. We have implicitly extremized with respect to the axion b_4 to get a negative sign in front of the second term as described below equation (3.9). It is obvious that in the limit

$$\tau_5 \to \infty \qquad \text{with} \qquad a_4\tau_4 = \ln \mathcal{V}\,, \tag{3.31}$$

the potential approaches zero from below as the middle term of equation (3.30) dominates. This is illustrated in figure 1 where we plot the numerical values of $\ln(V)$.

The location and properties of the AdS minimum may be found analytically. To capture the form of equation (3.30), we write

$$V = \frac{\lambda\sqrt{\tau_4}e^{-2a_4\tau_4}}{\mathcal{V}} - \frac{\mu}{\mathcal{V}^2}\tau_4 e^{-a_4\tau_4} + \frac{\nu}{\mathcal{V}^3}\,. \tag{3.32}$$

The axion field b_5 has been ignored as terms in which it appears are exponentially suppressed.

We regard $V = V(\mathcal{V}, \tau_4)$, and solve

$$\frac{\partial V}{\partial \mathcal{V}} = \frac{\partial V}{\partial \tau_4} = 0\,.$$

One may easily check that the first of these equations may be rearranged into a quadratic and solved for $\mathcal{V}$ to give

$$\mathcal{V} = \frac{\mu}{\lambda}\sqrt{\tau_4}e^{a_4\tau_4}\left(1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_4^{\frac{3}{2}}}}\right). \tag{3.33}$$

We also have

$$\frac{\partial V}{\partial \tau_4} = 0 \Rightarrow \frac{\lambda\mathcal{V}e^{-a_4\tau_4}}{\tau_4^{\frac{1}{2}}}\left(\frac{1}{2} - 2a_4\tau_4\right) - \mu\left(1 - a_4\tau_4\right) = 0\,. \tag{3.34}$$

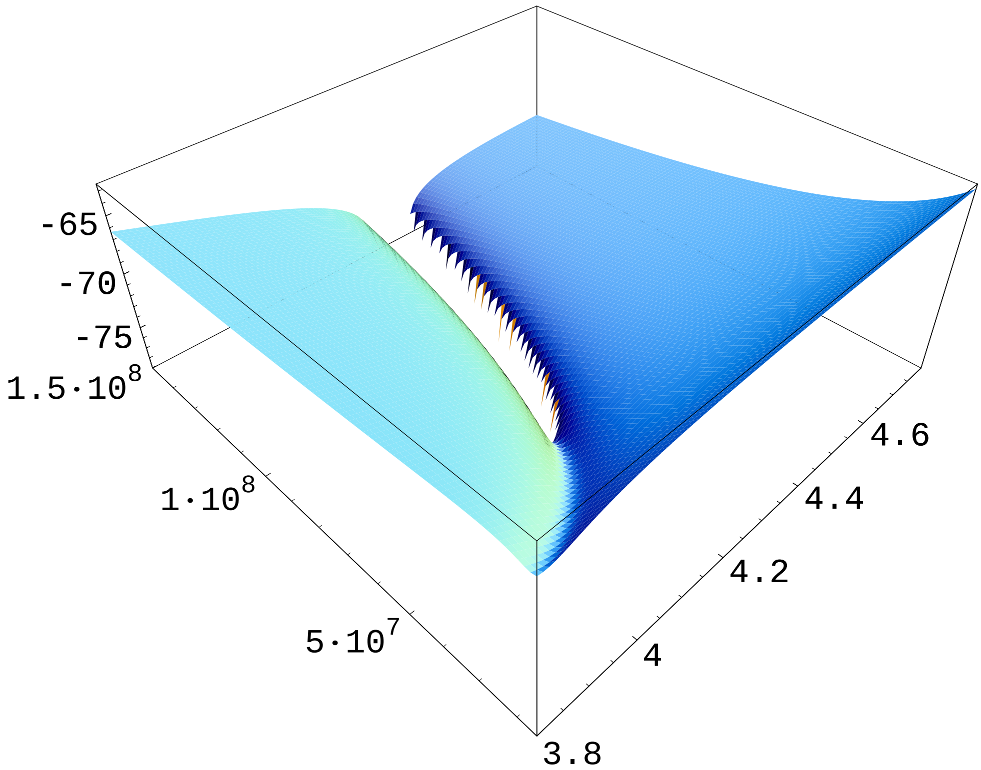

Figure 1: $\ln(V)$ for $P^4_{[1,1,1,6,9]}$ in the large volume limit, as a function of the divisors τ_4 and τ_5. The void channel corresponds to the region where V becomes negative and $\ln(V)$ undefined. As $V \to 0$ at infinite volume, this immediately shows that a large-volume minimum must exist. Here the values $W_0 = 20, A_4 = 1$ and $a_4 = 2\pi$ have been used.

We then use (3.33) to obtain an implicit equation for τ_4,

$$\left(1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu\tau_4^{\frac{3}{2}}}}\right)\left(\frac{1}{2} - 2a_4\tau_4\right) = (1 - a_4\tau_4)\,. \tag{3.35}$$

We do not need to solve this fully; as we require $a_4\tau_4 \gg 1$ to be able to ignore higher instanton corrections, we can use this to simplify (3.35) and solve for τ_4 and $\mathcal{V}$, obtaining

$$\begin{aligned} \tau_4 &= \left(\frac{4\nu\lambda}{\mu^2}\right)^{2/3}, \\ \mathcal{V} &= \frac{\mu}{2\lambda}\left(\frac{4\nu\lambda}{\mu^2}\right)^{1/3} e^{a_4\left(\frac{4\nu\lambda}{\mu^2}\right)^{2/3}}. \end{aligned} \tag{3.36}$$

For the potential of (3.32),

$$\lambda \sim a_4^2|A_4|^2\,, \qquad \mu \sim a_4|A_4W_0|\,, \qquad \text{and} \qquad \nu \sim \xi|W_0|^2\,.$$

We then have

$$\tau_4 \sim (4\xi)^{\frac{2}{3}} \qquad \text{and} \qquad \mathcal{V} \sim \frac{\xi^{\frac{1}{3}}|W_0|}{a_4A_4}e^{a_4\tau_4}\,. \tag{3.37}$$

This formula justifies our earlier claim that these vacua can generically be at exponentially large volume.

For the $P^4_{[1,1,1,6,9]}$ example,

$$\begin{aligned} \xi \;&=\; 1.31\,, \qquad \lambda = 3\sqrt{2}a_4^2|A_4|^2\,, \qquad \mu = \frac{1}{2}a_4|A_4W_0|\,, \qquad \nu = 0.123|W_0|^2 \\ &\Rightarrow \tau_4 \approx 4.11 + \text{ (small terms)}\,, \\ &\quad \mathcal{V} \approx 0.12\frac{W_0}{a_4A_4} \times e^{4.11a_4} + \text{ (small terms)}\,. \end{aligned} \tag{3.38}$$

These analytic results agree well with the exact locations of the minima found numerically, with the small error almost entirely due to the approximation made in solving equation (3.35). As discussed in section 3.1, the values of the Kähler moduli found above combine with the flux-stabilised complex structure moduli to give a minimum of the full potential (2.5).

We note that both the overall and divisor volumes are clearly larger than the string scale, and that as long as a_4 is not too small the gravitino mass $e^{K/2}|W|$ is well below the Planck scale. For $a_4 = 2\pi$ (as it is for D3-brane instantons), $A_4 = 1$, $W_0 = 10$, we obtain $\mathcal{V} \approx 3 \times 10^{10}$ in string units. The gravitino mass $m_{\frac{3}{2}} = e^{K/2}W$ is then given by $3(M_p g_s)/\sqrt{\mathcal{V}} \sim 7 \times 10^{12}$ GeV for $g_s = \frac{1}{10}$. Here M_p is the 4d Planck scale and we take $M_p \sim 10^{19}$ GeV. So we have an explicit realisation of the intermediate scale string scenario [31]. As discussed below, this is independent of the flux-induced value of W_0.

4. Discussion

We have shown that there exists a decompactification direction in moduli space along which nonperturbative D3-instanton effects dominate over the perturbative α'^3 corrections and therefore a large volume minimum is induced in a very general class of compactifications (for which $h_{12} > h_{11} > 1$).[8] There are several very interesting features about the above limit which may have important implications:

1. As exemplified by equation (3.37), the mechanism described here results in internal spaces that are exponentially large in string units. The largest such volumes arise for the case $a_4 = 2\pi$, when the non-perturbative dependence on τ_4 arises through D3-brane instantons. However, gaugino condensation with $a_4 = \frac{2\pi}{N}$ can also lead to large volume vacua. Having exponentially large volume $\mathcal{V}$ implies a realization of the large extra dimensions scenario in which the fundamental string scale is hierarchically smaller than the Planck scale since the string scale M_s and the 4d Planck scale are related by $M_s \sim M_p/\sqrt{\mathcal{V}}$. The ratio of the string scale to the 4d Planck scale is scanned over the different vacua in the sense that it depends explicitly on the fluxes

[8] In the special large volume limit that we have described, the potential will go to zero from below even if $h_{12} < h_{11}$, namely even if the parameter ξ in our analysis is negative. However, in this case the small volume behavior is harder to analyze, and so it is not clear if the non-supersymmetric minimum that we have found occurs.

through the stabilized Calabi-Yau volume. For the particular example discussed here we find an intermediate string scale $M_s \sim 10^{12}$ GeV, which has been claimed to have some phenomenological virtues [31].

2. The gravitino mass is given by $m_{\frac{3}{2}}^2 = e^K |W|^2$. As $e^K \sim \frac{1}{\mathcal{V}^2}$ and $\mathcal{V} \propto W_0$, this is unaffected by the value of W_0 arising from the flux choice. This gives the striking result that the discretuum of gravitini masses should be sharply peaked around one particular value. For the $P^4_{[1,1,1,6,9]}$ case, we have

$$m_{\frac{3}{2}} \approx 4 \times 10^9 \,\text{GeV}\,.$$

   Given the universality of the gravitino mass across the space of flux choices, it would be interesting to know whether this might also be universal or near-universal across the space of Calabi-Yau manifolds. This might result in the possibility of well-defined physical predictions from the discretuum of flux vacua.

3. The volume obtained scales as $\mathcal{V} \propto W_0$. Thus increasing W_0 increases the internal volume. This behaviour is opposite to that encountered in the KKLT scenario, where small values of W_0 are required for large volumes. It is also evident from equation (3.37) that in the case of D3-brane instantons with $a_4 = 2\pi$ essentially all values of W_0 will give an acceptable solution. Note that the results of Douglas and Denef [10] show that $e^{K_{cs}}|W_0|^2$ is uniformly distributed in the discretuum of flux choices and thus 'typical' choices of flux give rise to large values of W_0.

4. The mechanism described above relies on there being at least two Kähler moduli, which is of course the generic case. If all Kähler moduli appear non-perturbatively in the superpotential, then the mechanism above will stabilise all moduli. However, even if only some moduli appear in the superpotential, then as argued in section (3.1) the volume modulus will still be stabilised at exponentially large volume, and we also expect the flux-invariance of the gravitino mass to be unaltered. Further, as argued at the end of section 3.1, the stabilised values of the moduli will automatically represent a minimum of the full potential. Thus there is no need to perform the almost impossible check that all hundred-odd directions in moduli space are non-tachyonic.

5. The tuning is, in a precise sense, minimal. That is, the only tuning performed is that necessary to ensure that the dilaton is at weak coupling. This will always be necessary until strong-coupling string dynamics is better understood. In particular, there is no need to tune the value of W_0 — as emphasised above, any value will do.

We also note that as the solutions described above are not tachyonic, the increase in tachyons found near the conifold locus by Denef and Douglas [10] would not be applicable here. Therefore the natural accumulation of solutions around the conifold locus indicated by the Ashok-Douglas formula and checked in [16, 17] for particular cases, now holds for our solutions.

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Finally, we should consider effects that may destabilise the above behaviour. Higher order α' corrections are subleading in volume and so will not alter the above behaviour. More important would be other α'^3 corrections. The α'^3 correction used here descends from the $\mathcal{R}^4$ term in the ten-dimensional action, the coefficient of which is a modular form [32]. The coefficient ξ should then be promoted to a dilaton-dependent modular form. As we fix the dilaton using fluxes, this will not affect the qualitative behaviour described above although it may affect the numerical values. There may also be other α'^3 corrections descending from the flux terms in ten dimensions.[9] Little is known about these extra terms in this setting and it would be interesting to understand their effects.[10] In the presence of extra $1/\mathcal{V}^3$ corrections, the quantitative behaviour of our solution will change but the qualitative behaviour will remain. It is still an interesting open question to determine their exact effect. There is also the general concern about loop corrections after supersymmetry breaking. In principle these will be suppressed by powers of the dilaton field that can be always tuned to be small by the choice of fluxes, but a better understanding of these corrections would be worthwhile. Warping effects should be suppressed at large volume, but so are the α' corrections and it would be interesting to study their relative magnitudes. Finally, α' corrections due to localised sources should be considered.

We have worked in a particular limit of F-theory which can be interpreted as a type IIB orientifold. It would be of interest to address these questions in more general F-theory settings, which include D-brane moduli[33].

We have argued that, at least to leading order, supersymmetry is broken by the Kähler moduli and unbroken by the complex structure moduli. It should be possible to calculate the soft susy-breaking terms engendered by this minimum. Another interesting direction worth exploring is the study of this class of potentials in the context of brane/antibrane [34] or closed moduli (racetrack) inflation [35].

We hope to address some of these issues in the future.

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[9] We thank A. Sinha and M. Green for interesting conversations about this point.

[10] A preliminary estimate of these terms (e.g. the term proportional to G_3^8) shows them to scale as $1/\mathcal{V}^k$ with $k > 3$ in their contributions to the scalar potential. Thus they are suppressed compared to the terms we have considered. A detailed study of the effects of these extra terms lies beyond the scope of this paper. We thank A. Sinha for discussions on this point.

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